Copy 6
RM A53B11

NACA RM A53B11

MAY 5 1953

c. 2

RESEARCH MEMORANDUM

A THEORETICAL STUDY OF THE EFFECT OF CONTROL-DEFLECTION AND CONTROL-RATE LIMITATIONS ON THE NORMAL ACCELERATION AND ROLL RESPONSE OF A SUPERSONIC INTERCEPTOR

By Howard F. Matthews and Stanley F. Schmidt

Ames Aeronautical Laboratory
Moffett Field, Calif.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON
April 29, 1953

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

A THEORETICAL STUDY OF THE EFFECT OF CONTROL-DEFLECTION AND CONTROL-RATE LIMITATIONS ON THE NORMAL ACCELERATION AND ROLL RESPONSE OF A SUPERSONIC INTERCEPTOR

By Howard F. Matthews and Stanley F. Schmidt

SUMMARY

A theoretical study was made of the effect of limiting the deflection and rate of deflection of the control surface on the normal acceleration and roll response to step commands for a representative, automatically controlled, supersonic, tailless interceptor. The results of the study showed that the normal-acceleration and roll-response times decrease at a diminishing rate with increases in the limited rate of control motion; that for the chosen conditions little reduction in the response time of the normal acceleration or the roll response is made by increasing the limited rate of control motion beyond about 200° per second; and that the incremental decrease in the roll response time, due to increases in the control deflection limit, increases with increases in the limited rate of control motion. It is also noted that the rate limit of 200° per second is considerably in excess of the minimum required for piloted airplanes by the applicable Air Force specification.

INTRODUCTION

The flight speeds attainable by jet-propelled bombers flying at high altitudes lead to high closing speeds of the interceptor. These high closing speeds so shorten the time for tactical decision and action by the pilot that fully automatic control appears necessary for the tracking phase of the interception. Since the interceptor response times must be short for precision control of the required flight path, which may vary rapidly due to target motion, relatively rapid changes of the control position are required. However, since even an infinite rate of control movement will not reduce the response time to zero, due to control-deflection limits, it is evident that the selection of a design control rate must be a compromise between conflicting requirements based on the rapidity of airplane response and the practical limitations of the size and power of the control servo and associated equipment. It is of interest then to compute the effects of limiting the control motion on the response of a representative airplane, and this report presents the results of such an investigation.

The airplane characteristics used were similar to those of a tailless supersonic interceptor. Simplified control systems, two in pitch and one in roll, were examined for the effects of limiting the control deflection and the rate of control movement at a Mach number of 1.5 and a pressure altitude of 40,000 feet.

NOTATION

A_Z normal acceleration

b wing span, ft

C_L lift coefficient

C_l rolling-moment coefficient

C_m pitching-moment coefficient

c local chord, ft

$\bar{c}$ wing mean aerodynamic chord, $\dfrac{\int_0^{b/2} c^2 dy}{\int_0^{b/2} c\ dy}$, ft

g gravitational acceleration, ft/sec^2

I_y pitching moment of inertia, slug-ft^2

I_x rolling moment of inertia, slug-ft^2

i $\sqrt{-1}$

K gearing (gain)

m mass of airplane, slugs

p a variable introduced in the Laplace transformation

q dynamic pressure, lb/sq ft

S	wing area, ft^2
T	time constant, sec
t	time, sec
V	velocity, ft/sec
v	volts
y	spanwise station of local chord, c, ft
α	angle of attack
γ	flight path angle
δ	control deflection
θ	angle of pitch
ζ	damping ratio
φ	angle of roll
ω	angular frequency, radians/sec

Subscripts

A	accelerometer
a	aileron
e	elevator
r	rate gyro
I	in
L	limit
o	out
p	pendulum
s	servo

All angles are in radians unless otherwise noted. A $(\dot{\ })$ above a symbol represents the first derivative with respect to time. The symbols C_{L_α}, C_{m_δ}, $C_{l_{\dot\varphi}}$, ... represent $\partial C_L/\partial\alpha$, $\partial C_m/\partial\delta$, $\partial C_l/\partial\dot\varphi$, ..., etc. Other symbols which are combinations of aerodynamic parameters are defined in the report as they occur. It should be noted that the quantities of interest in the figures of the report represent amounts measured from the initial steady state.

AERODYNAMICS, CONTROL SYSTEMS, AND COMPONENTS

Aerodynamics

The pertinent mass and theoretical aerodynamic characteristics of the representative interceptor for the flight condition of a Mach number of 1.5 and a pressure altitude of 40,000 feet are listed in table I. The simplified, rigid-airplane equations of motion used are as follows:

Pitch (two degrees of freedom)

$$mV\dot{\gamma} = qS(C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e)$$

$$I_y\ddot{\theta} = qS\bar{c}(C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_{\dot\alpha}}\dot\alpha + C_{m_{\dot\theta}}\dot\theta)$$

Roll (single degree of freedom)

$$I_x\ddot{\varphi} = qSb(C_{l_{\delta_a}}\delta_a + C_{l_{\dot\varphi}}\dot\varphi)$$

The necessary aerodynamic transfer functions are derived in the usual manner from these equations and are as follows:

Pitch

$$\frac{A_{Z_0}}{\delta_e} = \frac{K_{A_Z}\left(1 + \frac{2\zeta_{\dot\gamma}}{\omega_{\dot\gamma}}p + \frac{1}{\omega_{\dot\gamma}^2}p^2\right)}{1 + \frac{2\zeta_n}{\omega_n}p + \frac{1}{\omega_n^2}p^2}$$

$$\frac{\dot\theta}{\delta_e} = \frac{K_{\dot\theta}(1 + T_{\dot\theta}p)}{1 + \frac{2\zeta_n}{\omega_n}p + \frac{1}{\omega_n^2}p^2}$$

$$\frac{A_{Z_o}}{\dot{\theta}} = \frac{\dfrac{K_{A_Z}}{K_{\dot{\theta}}}\left(1 + \dfrac{2\zeta_{\dot{\gamma}}}{\omega_{\dot{\gamma}}} p + \dfrac{1}{\omega_{\dot{\gamma}}^2} p^2\right)}{1 + T_{\dot{\theta}} p}$$

Roll

$$\frac{\varphi_o}{\delta_a} = \frac{K_\varphi}{p(1 + T_\varphi p)}$$

where

$$K_{\dot{\theta}} \approx \frac{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}}{-\tau C_{m_\alpha}} \qquad K_\varphi = -\frac{C_{l_{\delta_a}}}{C_{l_{\dot{\varphi}}}}$$

$$K_{A_Z} = K_{\dot{\theta}} \frac{V}{g} \qquad \zeta_{\dot{\gamma}} = -\frac{(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}})}{2\sigma\omega_{\dot{\gamma}}}$$

$$\omega_{\dot{\gamma}} = \left(\frac{C_{L_\alpha} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_\alpha}}{\sigma C_{L_{\delta_e}}}\right)^{1/2} \qquad \zeta_n = \frac{\sigma C_{L_\alpha} - \tau(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}})}{2\sigma\tau\omega_n}$$

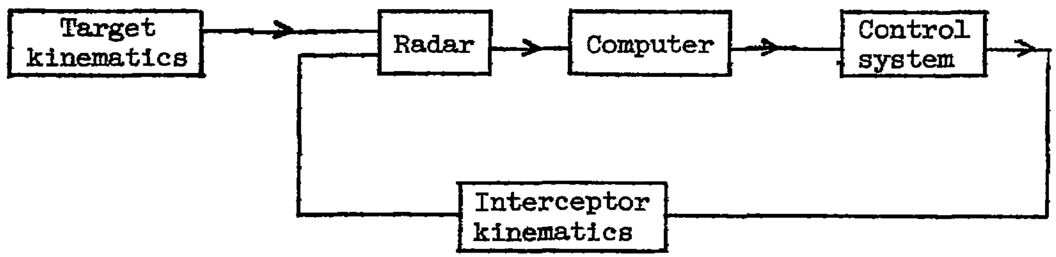
$$\omega_n \approx \left(\frac{-C_{m_\alpha}}{\sigma}\right)^{1/2} \qquad T_\varphi = -\frac{I_x}{qSb\, C_{l_{\dot{\varphi}}}}$$

$$T_{\dot{\theta}} = -\frac{C_{m_{\delta_e}}}{K_{\dot{\theta}} C_{m_\alpha}} \qquad \tau = \frac{mV}{qS}$$

$$\sigma = \frac{I_y}{qS\bar{c}}$$

Control Systems

In the automatic tracking phase of the flight of an interceptor, the aircraft control equipment usually attempts to govern the flight path in accordance with the requirements derived by a computer from information furnished by radar and other instrumentation. This may be schematically represented as follows:



Many control systems are possible. However, this investigation was restricted to the following particular systems, two in pitch and one in roll.

Pitch.- In figure 1 is shown a portion of a control system considered for this study. In this system the input voltage is proportional to the desired acceleration as determined by the computer. This input is compared with the output of an accelerometer in the usual manner for a closed-loop system to obtain the error signal. Error integration is provided to give equality between the input and output in the steady state, and error differentiation is used for obtaining the desired stability. As originally contemplated, the stabilizing network of figure 1 did not have the lag term $1/(1+0.1p)$ included, and the constants of the numerator were to be so proportioned as to cancel the denominator of the airplane transfer function. The explanation for the subsequent inclusion of the lag term is given in a later section of the report.

Since the over-all tracking system is a closed loop, it was believed that an investigation of the open-loop control system of figure 2, wherein the stability of the system is varied by feedback from the rate gyro and angular accelerometer, would be of interest. This system is termed open loop, since the output A_{Z_O} is not compared with the input from the computer to form an error signal and, in contrast to the previous system, the output depends on the gain of the system as well as the magnitude of the input signal.

In both systems, only a control-rate limiter was used since, as will be subsequently shown, the control deflections do not appreciably exceed the trim values.

Roll.- The roll control system considered is shown in figure 3. This is a position control closed-loop system, wherein the integration of the error signal necessary for zero steady-state error is provided by the aerodynamics. Stability of the system is changed by means of the roll rate gyro feedback loop. Limitations on the control deflection[1] in addition to the rate limit were applied in the simulator studies.

Components

The characteristics of the servos and measuring instruments as used in all the systems can usually be represented to a high degree of accuracy by a second-order transfer function, containing a gearing K, a natural frequency ω_n, and a damping ratio ζ, as illustrated in the following equation:

$$\frac{\text{output}}{\text{input}} = \frac{K}{1 + (2\zeta/\omega_n)\, p + (1/\omega_n^2)\, p^2}$$

However, if the natural frequency of the component is much higher than the aircraft short-period natural frequency (approximately 1.2 cycles per second in pitch) then the p^2 term in the denominator of the transfer function can be neglected with little error in the results. This simplification has been used for most of the components of the systems of this investigation.

Servos.- For a tailless aircraft the same controls are usually used for both pitch and roll, being deflected together for pitch control and differentially for roll. For this reason the same servo characteristics were used for both the pitch and roll systems and were assumed to be a natural frequency of 10 cps and a damping ratio of 0.35. These characteristics result in the following approximate first-order transfer function:

$$\frac{K_s}{1 + 0.01\, p}$$

Accelerometer and roll rate gyro.- For these two instruments a natural frequency of 20 cps and a damping ratio of 0.7 was assumed, resulting in an equivalent first-order time lag of approximately 0.01 second. The transfer functions are the following:

$$\frac{K_A}{1 + 0.01\, p} \qquad \text{accelerometer}$$

[1]A discussion of the manner in which the control-deflection limit was simulated in conjunction with the first-order servo is presented in the appendix.

$$\frac{K_r p}{1+0.01\,p} \qquad \text{roll rate gyro}$$

Other instruments.- The free gyro of the roll system and the rate gyro and angular accelerometer of the pitch system were assumed to have no time lags so that the transfer function becomes only a gearing. Actually, the latter two instruments do have small time lags but, to establish for comparative purposes the best possible response of the open-loop normal-acceleration control system, they were taken as zero.

METHOD OF ANALYSIS

The present system is nonlinear due to the limits placed on the control motion and, therefore, for rapid system optimization the use of some type of high-speed analogue computer is desirable. A high-speed electronic simulator available at the Ames Aeronautical Laboratory was used for this purpose. In this analogue computer the quantities of interest are transmitted as repetitive voltages to an oscilloscope for observation and, if desired, photographic recording.

To provide a consistent basis of comparison in pitch, the system parameters were adjusted to result in a 10-percent initial overshoot in the acceleration output response to a step g input with no undershoot of the steady-state value. For roll a deadbeat response was used. As a figure of merit, the time to within 10 percent of steady state was chosen for the pitch response and the time to within 10° (approximately 10 percent) was chosen for a 90° roll input.

Most of the results presented herein are in the form of "optimized" transient responses. These were obtained by varying the gearings and time constants of the particular systems to give the most rapid responses consistent with the conditions noted above. All variables for the data presented in the data figures of the report are listed in table II.

RESULTS AND DISCUSSION

Pitch

Closed loop.- As noted previously, the constants of the stabilizing network without the time lag were intended to cancel the denominator of the airplane transfer function, with the objective of obtaining a rapid response. Figure 4 illustrates this condition for a 4g step input with the rate of control motion limited to 100° per second. It is apparent that the behavior of the aircraft control system is unsatisfactory, primarily because of the poor damping. Increasing the gain of the system leads to a small amplitude, relatively undamped, high-frequency oscillation superimposed upon a response similar to that of figure 4.

Figure 5 shows the best response obtainable with the given form of stabilizing network for the same input and control-rate limit as of figure 4, but with the time constants not being equal to those of the denominator of the airplane transfer function. The response is improved but the damping is still undesirably low.

From a study of the elevator motion of figures 4 and 5, it was reasoned that the motion required for a good response was first to move at maximum rate in a direction toward the steady-state value, then to turn and move in the opposite direction at maximum rate as the error signal decreases to produce a braking force, and finally to approach the trim value as the error integral signal approaches its steady-state value. Since the magnitude and duration of the derivative of the error signal primarily determine the initial movements of the elevator, it was believed that increasing the duration of the error rate signal by means of the addition of a time lag to the stabilizing network would lead toward obtaining the described motion of the elevator. This hypothesis was substantiated by the results of figure 6. The time histories shown in this figure are the result of adding a 0.1-second time lag[2] to the stabilizing network and adjusting the constants to give the desired output for the condition of a 4g input and a rate limit of 100° per second of the control.

The time lag of 0.1 second for the stabilizing network proved to be close to optimum and was used in the remainder of the investigation of this system. The solid curve of figure 7 shows the result of optimizing the response for a limited rate of control motion varying from 50° to 300° per second. Figure 8 shows the response in acceleration and control motion at these two limits. Note that the control deflection exceeds only slightly the trim value, and therefore a practical deflection limit has no effect on the results. The result of the solid curve of figure 7 indicates that little advantage in speed of response is gained by exceeding a rate of control motion of about 200° per second.

It is also of interest to consider the output response to different input magnitudes since they are not independent if the system is nonlinear. Shown in figure 9 are the results for a rate limit of 100° per

---

[2]The addition of the time lag makes the error stabilizing network physically realizable, since the denominator and numerator are of equal order.

second of three magnitudes of the input: 4g, 2.5g, and 1g with the system optimized for the 4g condition. In figure 10 are shown similar responses but with the system optimized for the 2.5g input. For either case the changes in the response due to the magnitude of the input appear to be unimportant.

Open loop.- A similar study of the effect of limiting the rate of the control motion on the response of the aircraft in pitch was made for the system of figure 2, and the results are summarized as the dotted curve of figure 7. It is apparent that the open-loop response is characterized by an almost constant reduction in response time of 0.05 second over that of the closed-loop system. The more rapid response was found to arise from larger control deflections, the trim value being exceeded for rates greater than 150° per second. Since the open loop does not have the lags associated with the error integration and the instruments in the stabilizing loop, the results noted in figure 7 represent close to the maximum possible acceleration response of the airplane and so may be used as a basis for judging other systems.

Roll

An investigation similar to that made on the normal-acceleration response was made for the time to roll to 90°. The comparison criterion was the minimum time to get within 10° of steady state and, contrary to the pitch response, control-deflection limiting was an important variable. The results of limiting both the rate and deflection of the control motion on the response in roll are summarized in figure 11. The dashed curve in this figure was obtained by reducing the control effectiveness 25 percent and is indicative of the possible effect of aeroelasticity. From this figure it is also evident that at low control-deflection limits, not as much gain in rolling performance can be obtained by increasing the rate of control motion as at the high deflection limits. At the higher control-deflection limits the same general conclusion reached for the normal-acceleration response also applies; that is, that a significant gain in performance is realized up to approximately 200° per second rate limitation. In figure 12 are shown the rolling-response and control-deflection time histories for the limiting rates of control deflection of 50° and 300° per second for a control limit of ±15°.

In figure 13 are shown the effects of increasing the magnitude of the input on the system optimized for an input of 45°. Two control-deflection limits are considered, 15° and none, for a rate limit of 150° per second. It is evident from the figure that increasing the input leads toward an oscillatory response, in fact, for the no-deflection-limit case the response becomes unstable for inputs greater than about 230°. However, the control deflections for this magnitude of input are impractically large having a maximum value of approximately 48°. Figure 13 also indicates the stabilizing effect of the control-deflection limiter. An examination of the block diagram of the control system (fig. 3), shows that the stabilizing effect is obtained by the limiter acting as an attenuator to the system. The stabilizing action of the control-deflection limiter is shown, perhaps more clearly, in figure 14 in which the control system is optimized at a rate limit of 50° per second for a 20° input, so that the control-deflection limit of 15° is not reached. At increasingly larger inputs, the response becomes more oscillatory until the control motion reaches the deflection limit, after which the oscillatory character of the response remains substantially unchanged with further increases in the magnitude of the input. For the same control-deflection limits, the effect of the magnitude of the input is reduced at higher control rates. For inputs smaller than the optimized input, the response, in all cases, is more deadbeat.

General Comments

Reference 1, the Air Force specification on flying qualities for piloted airplanes, requires that for a power-operated or power-boost control system, the system should be capable of moving the control surfaces at rates of 50° per second or more. It appears then from the results of this investigation that, for aircraft which have both manual- and automatic-control phases of flight, the requirements of the latter phase will design the rate of control motion. This is not surprising since the reason for the automatic-control system is to obtain greater speed and accuracy in the tracking type of task and, hence, higher component performances are required.

The results of this investigation may be quantitatively altered (the trends should remain relatively unchanged) if, in the selection of the system characteristics, consideration is given to system "noise" and to the sensitivity of system stability to changes in component characteristics.

CONCLUSIONS

A theoretical investigation of the response in normal acceleration and roll to step inputs has been made for a representative supersonic tailless interceptor with several typical pitch and roll automatic control systems. Limitations were placed on the rate of control motion and on the control deflection. From the investigation the following conclusions may be drawn:

1. The normal-acceleration and roll response times decrease at a diminishing rate with increases in the limited rate of control motion.

2. The incremental decrease in the roll-response time, due to increases in the control-deflection limit, increases with increases in the limited rate of control motion.

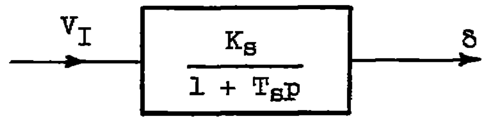
3. For the conditions of this study, little reduction in the response time of the normal acceleration or the roll response is made by increasing the limited rate of control motion beyond about 200° per second. This rate is considerably in excess of the minimum required for piloted airplanes by the applicable Air Force specification.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif.

APPENDIX

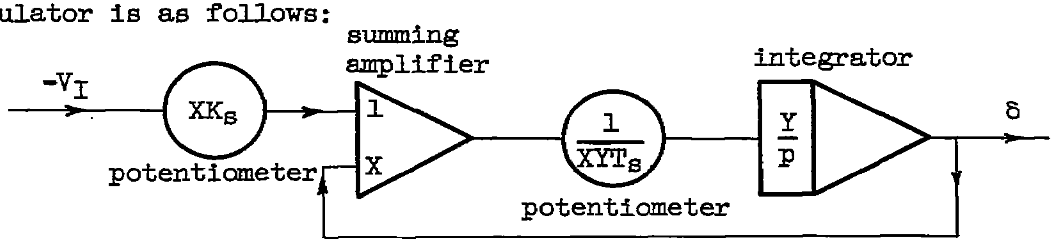
SIMULATION OF A FIRST-ORDER SERVO WITH A CONTROL-DEFLECTION LIMIT

It is frequently the practice in simulator studies involving control systems to represent the control servo in block diagram form by the first-order transfer function:



Sketch (a)

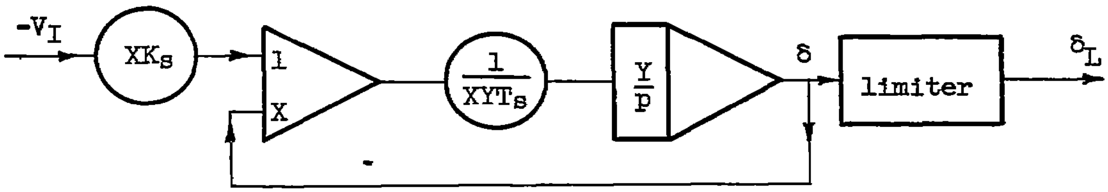
The representation of this transfer function on an analogue computer or simulator is as follows:



Sketch (b)

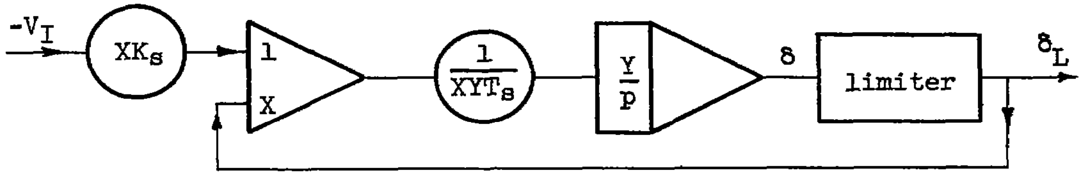
The choice of X and Y are based upon the construction and limitations of the components of the simulator (an X of 5 and a Y of 20 were used in the foregoing study).

Occasionally it is also necessary to add a control-deflection-limit condition to the servo to simulate the action of control stops. A perusal of the literature indicates that this condition commonly has been simulated incorrectly as



Sketch (c)

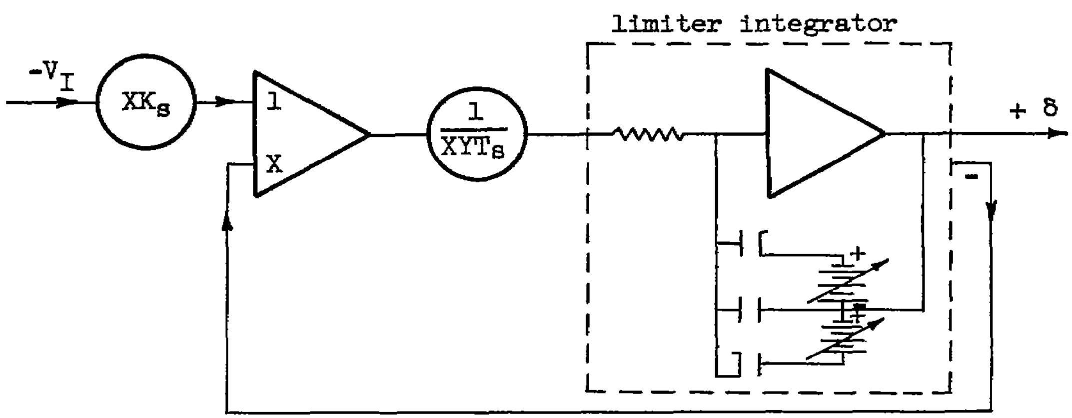
or



Sketch (d)

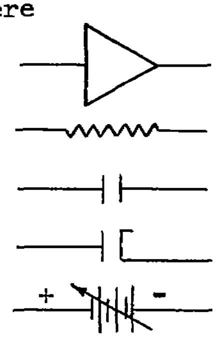
The error in these instances arises in neglecting to observe that when δ reaches the limit, the integrator, which duplicates the function of the hydraulic ram, should stop integrating. Neither of the above simulator representations accomplishes this but, rather, acts somewhat as a variable backlash device.

One means of correctly simulating the action of control stops is by the use of relays. Relays, however, introduce lags when used at high frequencies and could not be tolerated on the high-speed simulator used in this study. A successful simulator setup is as follows:



Sketch (e)

where



a high gain d.c. operational amplifier

a resistor

a capacitor

a diode

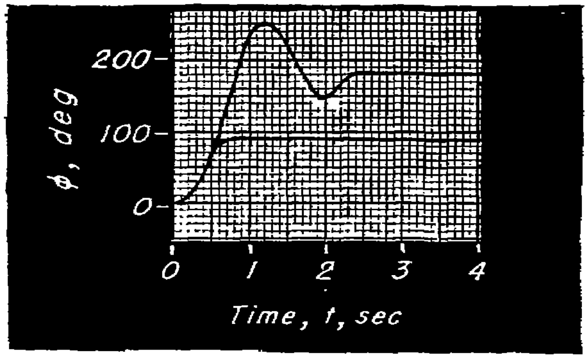
a variable voltage battery or power supply

This installation properly simulates a first-order servo with a control-deflection limit, since the integrator stops when its output reaches the limit.

The effect of incorrectly simulating the action of control stops can be judged by a comparison of figure 13(a) of the report with the following:

It is evident that using the incorrect simulation of the action of control stops leads to the erroneous conclusion that the oscillatory characteristics of the roll response is markedly influenced by the magnitude of the input.

It should be noted that a comparable difficulty does not exist in imposing a limit on the rate of control motion as it is correctly simulated by a voltage limiter placed ahead of the integrator.



Effect of roll input. (Optimized for φ of 90°. $\delta_L = \pm 15°$. $\dot{\delta}_L = 150°/\text{sec}$. Incorrect simulation technique, sketch (d).)

Sketch (f)

REFERENCE

1. Anon.: Flying Qualities of Piloted Airplanes. Spec. No. 1815-B, U. S. Air Force, June 1, 1948.

TABLE I.- GEOMETRIC, MASS, AND AERODYNAMIC PARAMETERS OF THE INTERCEPTOR AT A MACH NUMBER OF 1.5 AND AN ALTITUDE OF 40,000 FEET

[Center of gravity at 27.5-percent $\bar{c}$]

Geometric and mass characteristics		Aerodynamic parameters		Transfer function parameters	
b	36.7	$C_{L_{\alpha}}$	2.46	$\zeta_{\dot{\gamma}}$	-0.0326i
$\bar{c}$	22.9	$C_{L_{\delta_e}}$	.32	ζ_n	.0879
m	710	$C_{m_{\alpha}}$	-.53	$\omega_{\dot{\gamma}}$	10.65i
S	650	$C_{m_{\delta_e}}$	-.21	ω_n	7.38
I_y	89,400	$C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}}$	-.0067	$K_{\dot{\theta}}$(1/sec)	-.258
I_x	13,600	$C_{l_{\delta_a}}$	-.063	K_{A_Z}(g/deg)	-.204
		$C_{l_{\dot{\varphi}}}$	-.0026	K_{φ}(1/sec)	-29.0
		q	617	$T_{\dot{\theta}}$(sec)	1.55
		V	1456	T_{φ}(sec)	.35

TABLE II.- SYSTEMS CHARACTERISTICS

(a) Pitch

Closed loop (K_A = 1 volt/g)				
Figure	$\dot{\delta}_{e_L}$, deg/sec	K_s, deg/volt	ζ	ω, 1/sec
4	100	-6.5	0.0879	7.38
5		-18.4	.545	12.3
6		-11.2	.107	6.35
7	50	-9.4	.121	5.79
	75	-10.7	.093	6.23
	100	-11.2	.107	6.35
	150	-12.7	.104	6.71
	200	-13.8	.107	6.78
	300	-14.6	.108	7.08
8	50	-9.4	.121	5.79
	300	-14.6	.108	7.08
9	100	-11.2	.107	6.35
10	100	-14.0	.119	6.90

Open loop (K_s = -1 deg/volt)				
Figure	$\dot{\delta}_{e_L}$, deg/sec	K_r, $\frac{\text{volts}}{\text{deg/sec}}$	T_r, sec	V_i, volts
7	40	0.329	-0.054	21.5
	55	.350	-.057	21.6
	75	.358	-.063	21.6
	100	.370	-.065	21.7
	150	.358	-.068	21.6
	225	.352	-.076	21.6
	300	.344	-.081	21.6

TABLE II.- SYSTEMS CHARACTERISTICS - Concluded

(b) Roll

[K_p = 1 volt/deg]

Figure	Gearing	δ_{a_L} °	$\dot{\delta}_{a_L}$ deg/sec 40	50	60	75	100	150	200	300
11	K_s	None	0.452	0.524	0.520	0.588	0.660	0.800	0.892	1.16
	K_r		.349	.314	.311	.314	.275	.242	.217	.200
	K_s	20	.452	.524	.520	.584	.752	1.03	1.40	1.68
	K_r		.349	.314	.311	.287	.235	.185	.156	.130
	K_s	15	.452	.524	.580	.780	.964	1.39	1.76	2.20
	K_r		.349	.314	.261	.225	.192	.154	.139	.120
	K_s	10	.588	.720	.856	.984	1.42	2.19	2.31	2.94
	K_r		.255	.229	.205	.182	.169	.139	.132	.122
	K_s	15 With reduced $C_{l_{\delta_a}}$	.540	.672	.796	.972	1.20	1.94	2.44	2.60
	K_r		.365	.300	.272	.231	.199	.164	.145	.120
12	K_s	15	- - -	.524	- - -	- - -	- - -	- - -	- - -	2.20
	K_r			.314						.120
13	K_s	15	- - -	- - -	- - -	- - -	- - -	1.25	- - -	- - -
	K_r							.155		
	K_s	None	- - -	- - -	- - -	- - -	- - -	1.16	- - -	- - -
	K_r							.200		
14	K_s	15	- - -	1.05	- - -	- - -	- - -	- - -	- - -	- - -
	K_r			.202						

Note: K_s, deg/volt; K_r, $\frac{\text{volts}}{\text{deg/sec}}$

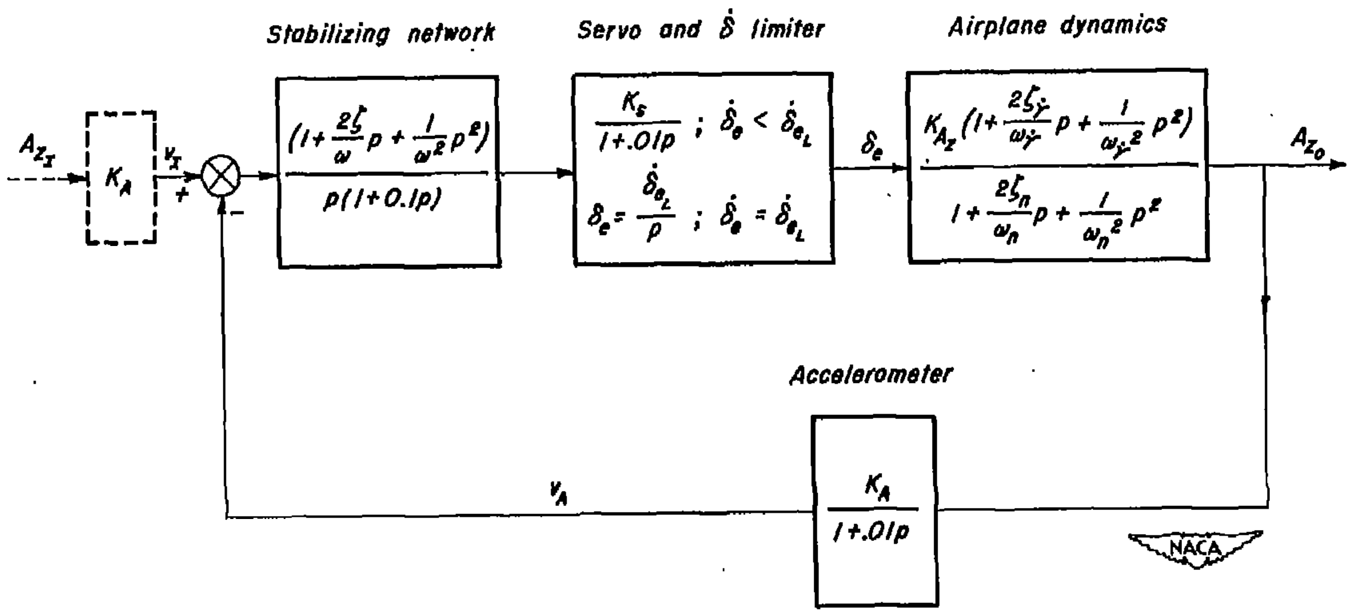


Figure 1.– Normal–acceleration control system; closed loop with error integration and rate stabilization.

[redacted]

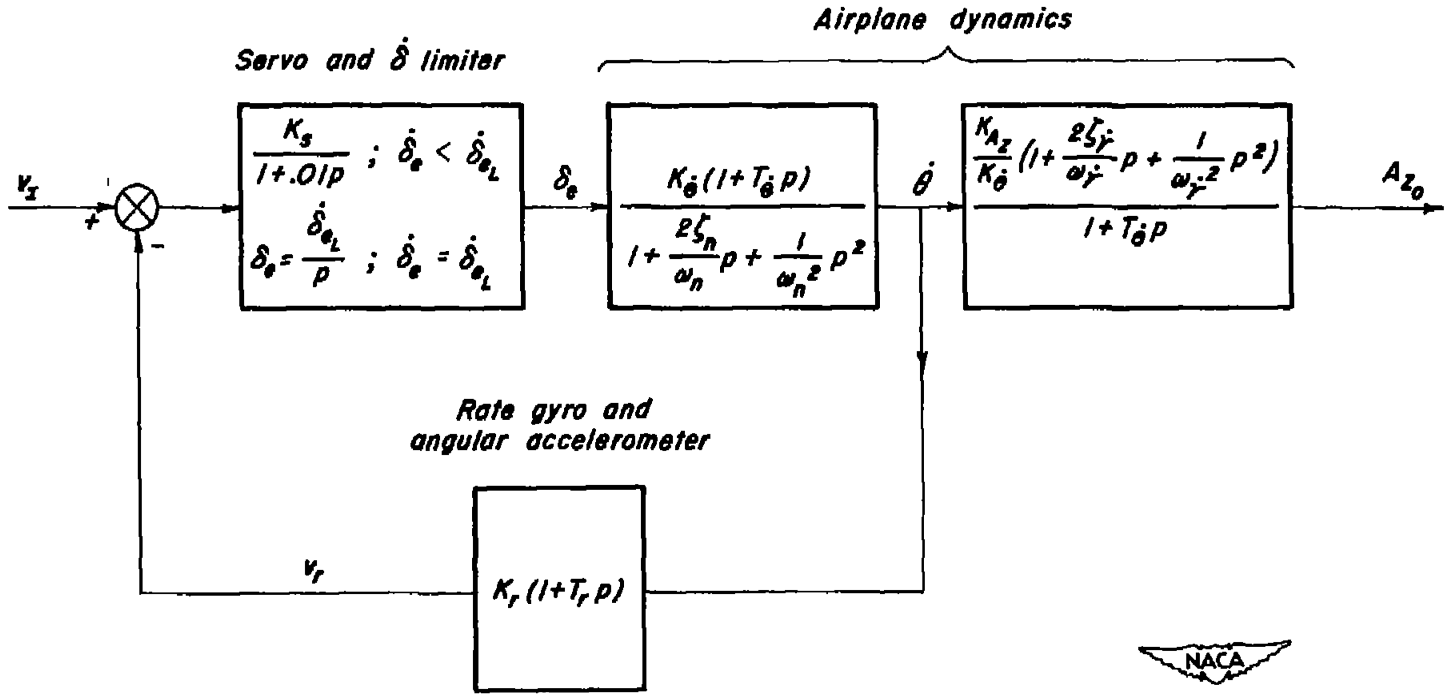
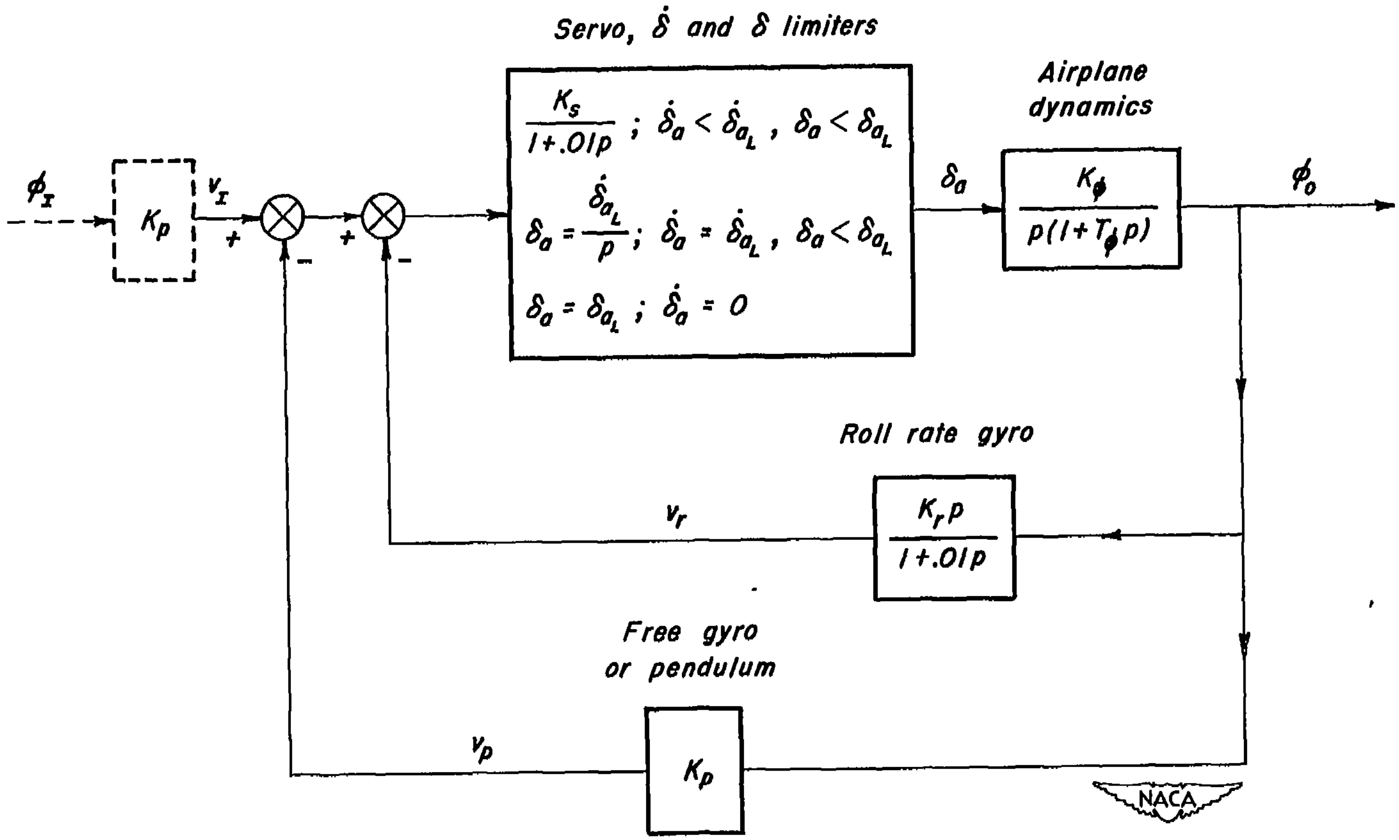


Figure 2.– Normal–acceleration control system; open loop with pitch rate stabilization.

[redacted]

[redacted]



[redacted]

Figure 3.– Roll control system.

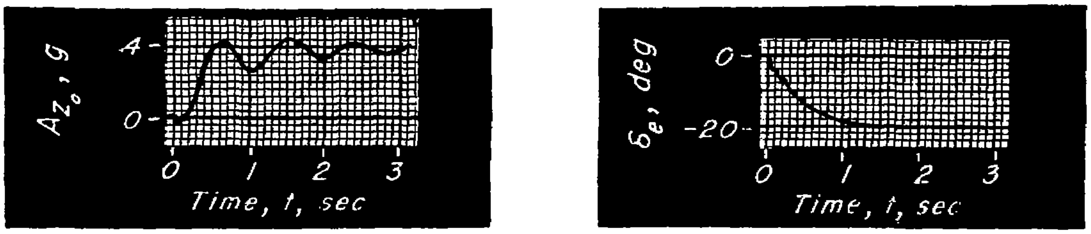


Figure 4.- Optimum response for stabilizing network (no time lag) canceling denominator of airplane transfer function; $\dot{\delta}_{e_L} = 100^\circ$/sec.

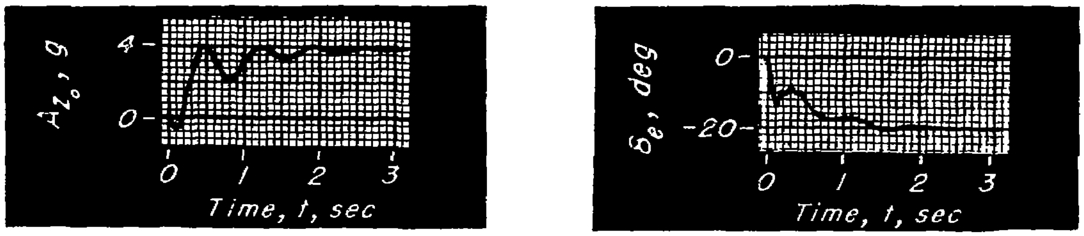
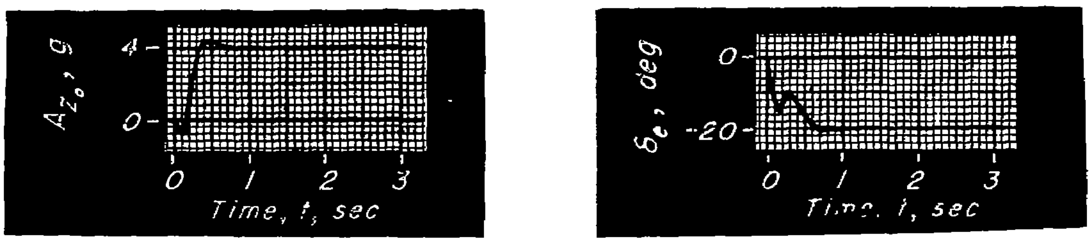


Figure 5.- Optimum response with given form of stabilizing network (no time lag); $\dot{\delta}_{e_L} = 100^\circ$/sec.



NACA
A-17908

Figure 6.- Optimum response with 0.1-second time lag added to the given form of stabilizing network; $\dot{\delta}_{e_L} = 100^\circ$/sec.

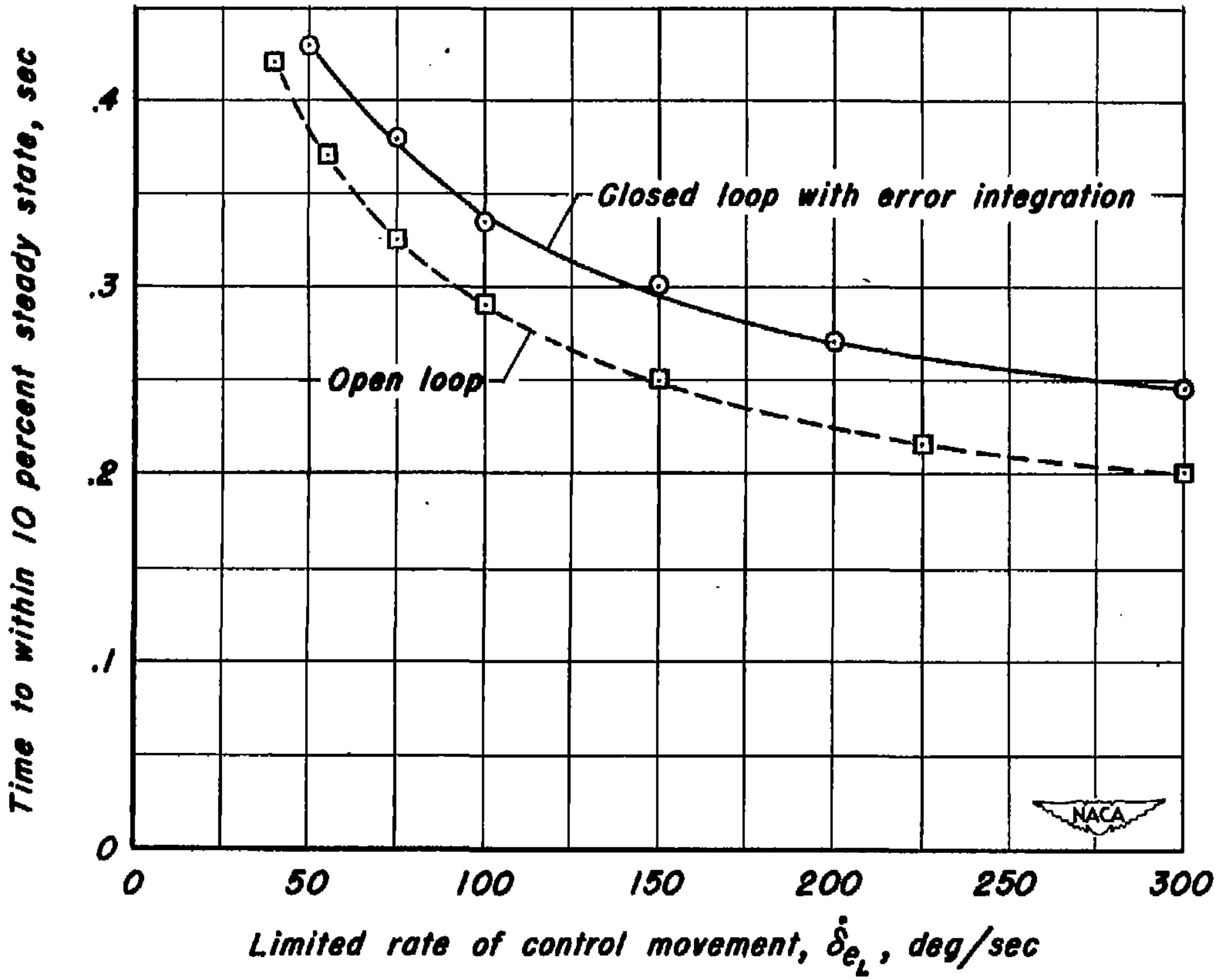


Figure 7.– Effect of limited rate of control movement on the normal acceleration response. $A_{z_0} = 4g$.

Figure 8.- Optimum responses for $\dot{\delta}_{e_L}$ of 50°/sec and 300°/sec.

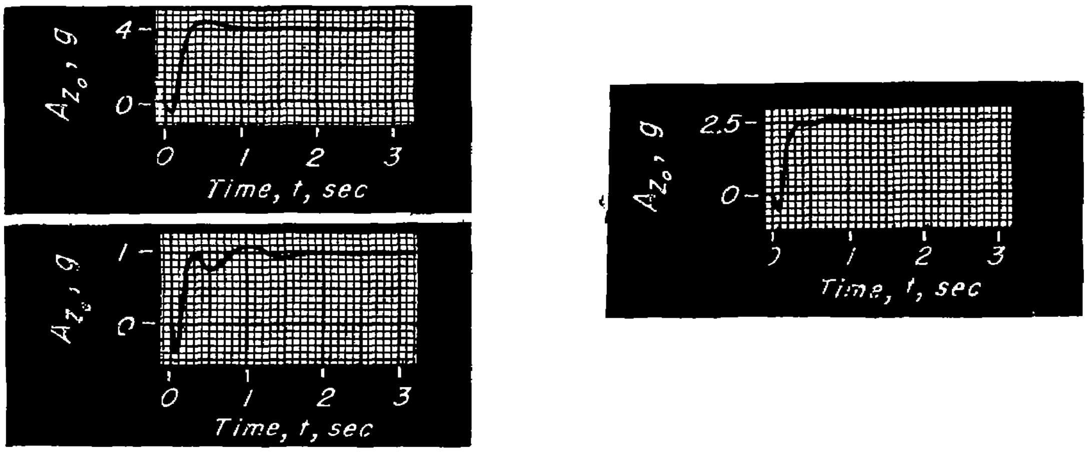
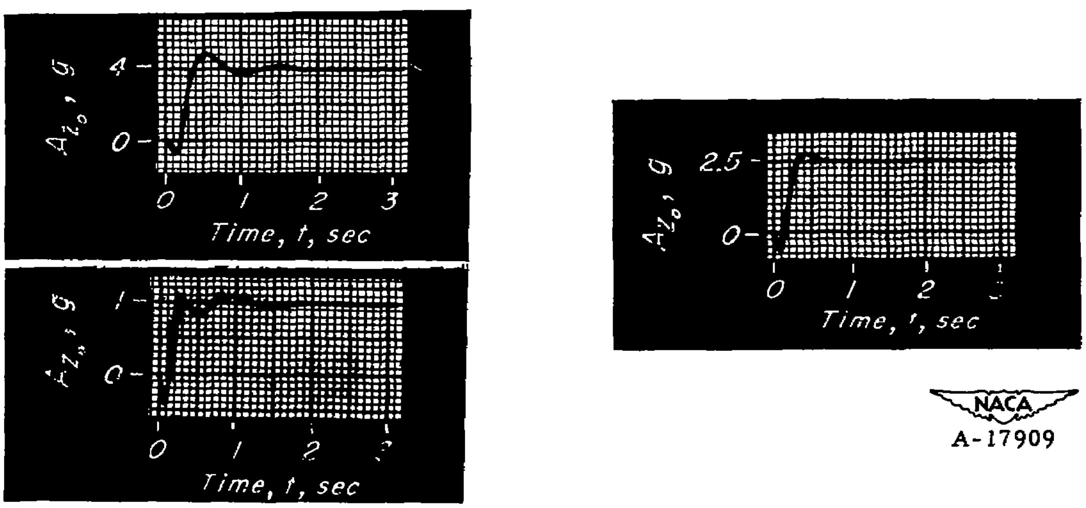


Figure 9.- Effect of input. Optimized for 4g and $\dot{\delta}_{e_L}$ of 100°/sec.



A-17909

Figure 10.- Effect of input. Optimized for 2.5g and $\dot{\delta}_{e_L}$ of 100°/sec.

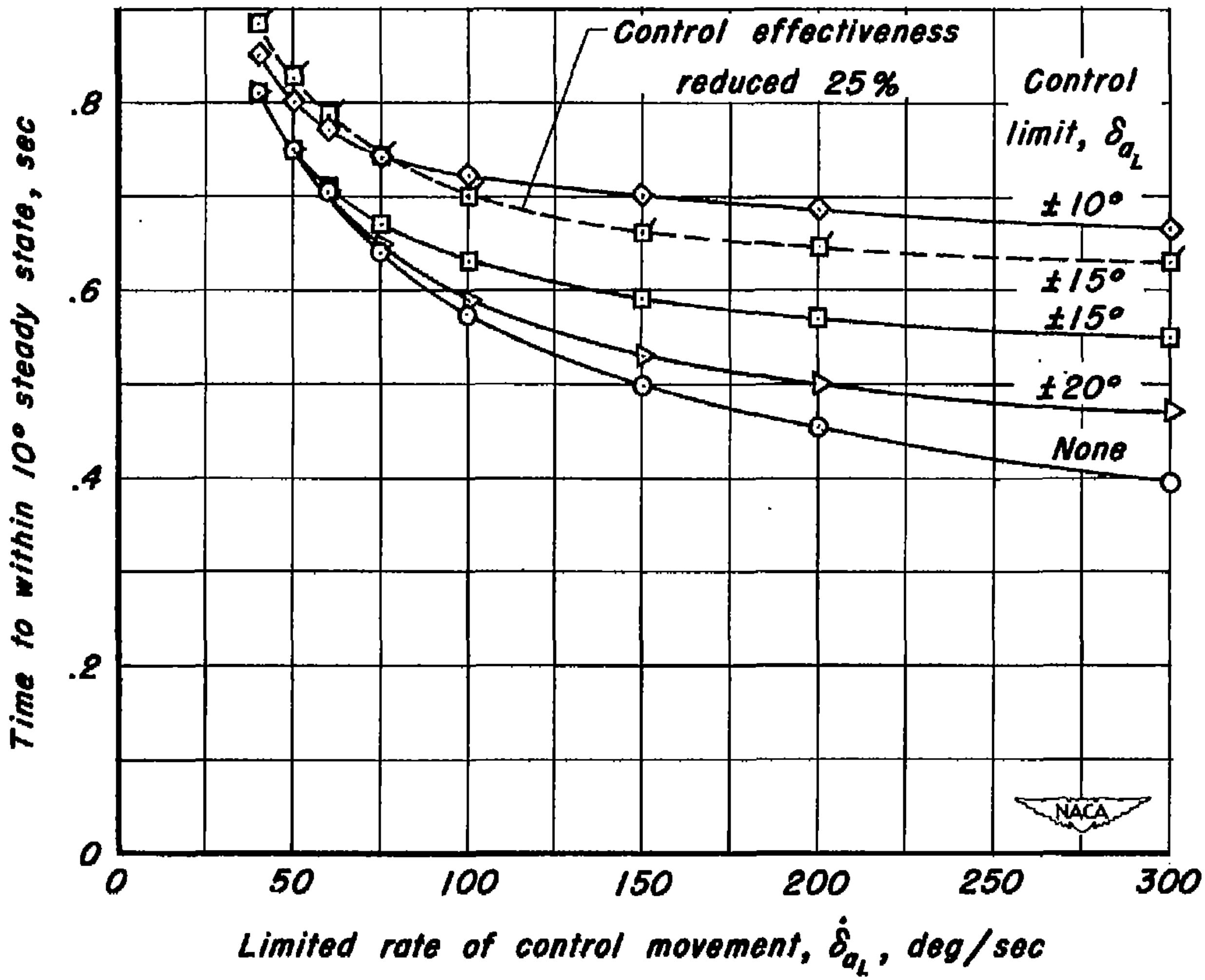


Figure 11.— Effect of limiting control deflection and rate of control movement. ϕ_0 = 90°.

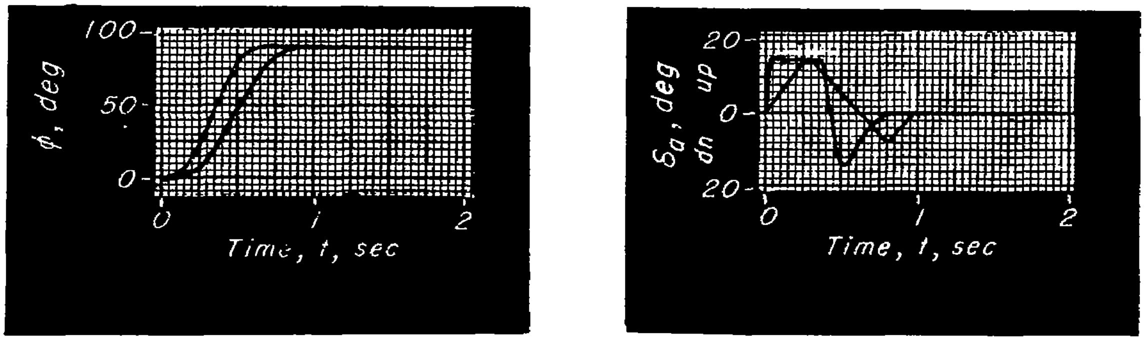
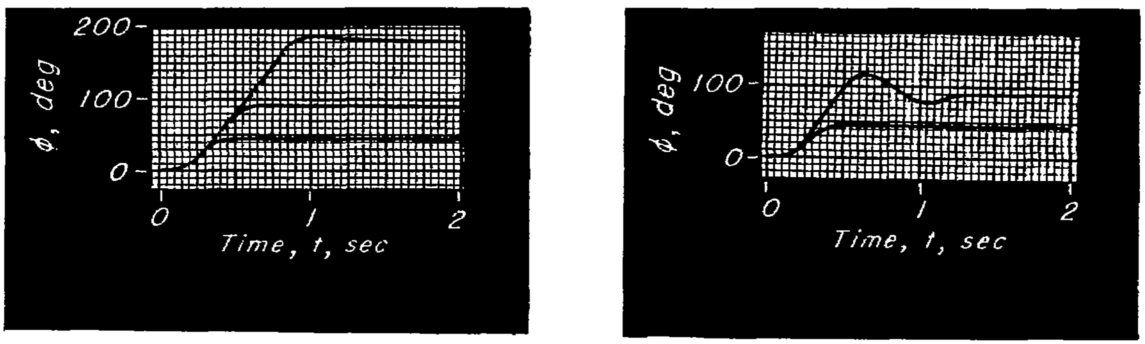


Figure 12.- Optimum responses for $\dot{\delta}_{a_L}$ of 50°/sec and 300°/sec; $\delta_{a_L} = \pm 15^\circ$.



(a) $\delta_{a_L} = \pm 15^\circ$

(b) δ_{a_L} = none

Figure 13.- Effect of input. Optimized for φ of 45°; $\dot{\delta}_{a_L} = 150^\circ$/sec.

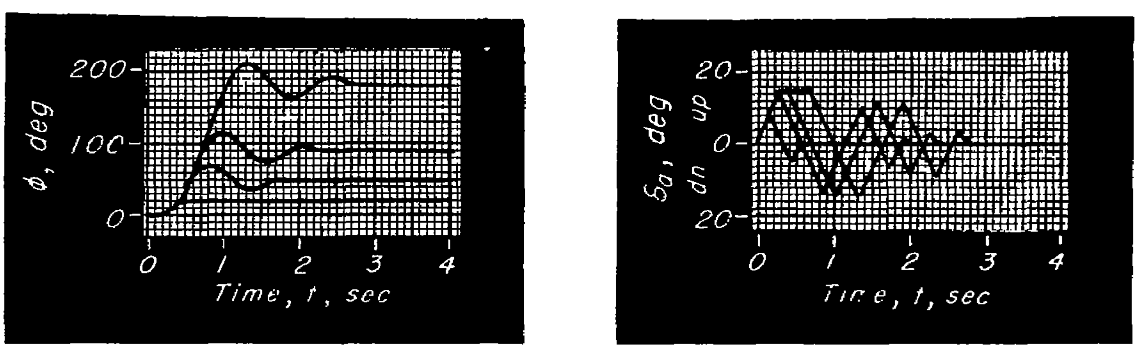


Figure 14.- Effect of input. Optimized for φ of 20°. $\dot{\delta}_{a_L} = 50^\circ$/sec, $\delta_{a_L} = \pm 15^\circ$.

NACA
A-17910

NACA-Langley - 4-29-53 - 400